Improving Online Algorithms via ML Predictions

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Table of Contents

Introduction

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- **Online Algorithms (OA)**
- Algorithms that make decisions one request at a time.
- Serve an sequence of item requests σ.
- Usually Oblivious adversary model generates arbitrary sequence.
- Online: No knowledge of future requests.
  - Given a time index t ordering the sequence, knowledge is available $\forall s \leq t$
- Research focus on *competitive analysis*.

$$ALG(\sigma) \leq c \cdot OPT(\sigma) + \alpha$$

- **Empirical** performance is **usually better** than suggested by competitive analysis.

Introduction

- OA are designed considering worst-case scenarios.
- **Can we improve the quality of OA if we have more information?**
  - Implication: weaker adversary.
- It is possible if predictions are available.
  - Reduced uncertainty.
- Competitive analysis no longer directly applies $\Rightarrow$ New framework.
- The paper explains through examples a method of extending OA whit predictions.
  - Skii rental.
  - Non-clairvoyant job scheduling.

Introduction - What to expect

What is the paper about:

- Examples on a method for extending online algorithms when predictions are available.
- A framework for its analysis.

What is the paper **not** about:

- The list update problem.
- Any particular cost model.
- Machine Learning – Predictions instead, overlap but not necessarily the same.

Framework

Framework

Predictions

- Uncertainty is addressed by making predictions.
- Assume predictor P s.t. makes an estimation y of a yet unknown value x.
- Let $\eta = \eta(x, y)$ be the **predictor error**.
  - **Problem dependent**.
  - $\eta \geq 0$.
  - If $x = y$ then $\eta = 0$.
- What if the error is large?
- If predictions are used naively, the end result could be arbitrarily bad!

Concepts

- Traditional algorithms: Competitive analysis, assumes worst-case scenarios.
  - Let c be the competitive ratio $c = \frac{ALG(\sigma)}{OPT(\sigma)}$
- Idea: express c as a function of predictor error η: $c(\eta)$.

Definition: γ-robustness

ALG is γ-robust if $\exists \gamma$ s.t. $\forall \eta$, $c(\eta) \leq \gamma$

- Measures algorithm performance under bad predictions.
- Smaller γ is better.

Definition: β-consistency

ALG is β-consistent if $c(0) = \beta$

- How well the algorithm does with perfect (errorless) predictions.
- Smaller β is better.

Design

Key design idea:

- **Hyperparameter**: Introduce an hyperparameter $\lambda \in (0, 1)$ into the online algorithms.
- λ: **"distrust in the prediction"**
  - λ near 0: Complete trust in the prediction.
  - λ near 1: Complete distrust in the prediction.
- **Intuition: Control with λ the impact of considering the prediction on the algorithm.**
- Trade-off between robustness and consistency.

Skii rental

Skii Rental

Skii rental

- Problem:
  - A skier can rent skiis at cost 1 per day or buy them at cost $b > 1$ and skii for free from then on.
  - Its objective is to minimize the cost.
  - The uncertainty is on the number of skiing days x.
  - Predicted number of skiing days: y.
  - Prediction error: $\eta = |x - y|$
- Optimal deterministic OA: **break-even**:
  - Rent on the first $b - 1$ days, buy on day b.
  - 2-competitive (tight)
- There is also a $\frac{e}{1-e}$ randomized OA.

Simple consistent non-robust algorithm

Algorithm 1 A simple 1-consistent algorithm

1: **if** $y \geq b$ **then**
2: Buy at the first day.
3: **else**
4: Keep renting for all the skiing days.
5: **end if**

Proof:

- $y \geq b, x \geq b \Rightarrow ALG = b = OPT$
- $y < b, x < b \Rightarrow ALG = x = OPT$
- $y \geq b, x < b \Rightarrow ALG = b \leq x + y - x = x + \eta = OPT + \eta$
- $y < b, x \geq b \Rightarrow ALG = x < b + x - y = b + \eta = OPT + \eta$ □.
- ∞-robust (bad!)

Deterministic algorithm

Algorithm 2 A deterministic robust and consistent algorithm

1: **if** $y \geq b$ **then**
2: Buy at the start of day $\lceil \lambda b \rceil$.
3: **else**
4: Buy at the start of day $\lceil b/\lambda \rceil$.
5: **end if**

- Competitive ratio: $\min\{\frac{1+\lambda}{\lambda}, (1+\lambda) + \frac{\eta}{(1-\lambda)OPT}\}$.
- $(1 + 1/\lambda)$-robust.
- $(1 + \lambda)$-consistent.

Analysis –Proof outline

Case 1

- If $y \geq b$ then buy the skis on day $\lceil \lambda b \rceil < b$.
  - If $x \geq \lceil \lambda b \rceil$ then $ALG = b + \lceil \lambda b \rceil - 1 \leq \left(\frac{1+\lambda}{\lambda} \right) \lambda b$, $OPT \geq \lceil \lambda b \rceil \Rightarrow c \leq \frac{1+\lambda}{\lambda}$

- If $y < b$ then buy on day $\lceil b/\lambda \rceil$. Worst case ratio happens if $x = \lceil b/\lambda \rceil$.
  - $ALG = b + \lceil \lambda b \rceil - 1 \leq \left(\frac{1+\lambda}{\lambda} \right) \lambda b$
  - $OPT = b \Rightarrow c \leq \frac{1+\lambda}{\lambda}$

Analysis –Proof outline

Case 2

If $y \geq b$ then buy the skis on day $\lceil \lambda b \rceil < b$,

- $\forall x < \lceil \lambda b \rceil \Rightarrow ALG = OPT = x$
- $\forall x \geq \lceil \lambda b \rceil \Rightarrow ALG = b + \lceil \lambda b \rceil - 1 \leq (1+\lambda)b \leq (1+\lambda)(OPT + \eta)$

If $y < b$ then buy the skis on day $\lceil \lambda b \rceil < b$,

- $\forall x < b \Rightarrow ALG = OPT = x$
- $\forall x \in (b, \lceil b/\lambda \rceil) \Rightarrow ALG = x \leq y + \eta < b + \eta = OPT + \eta$
- $\forall x \geq \lceil b/\lambda \rceil$
  - $\eta = x - y > b/\lambda - b = (1-\lambda)b/\lambda$
  - $\Rightarrow ALG = b + \lceil b/\lambda \rceil - 1 \leq b(1 + 1/\lambda) < b + (\frac{1}{1-\lambda})\eta = OPT + (\frac{1}{1-\lambda})\eta$

$$\Rightarrow ALG \leq (1+\lambda)OPT + (\frac{1}{1-\lambda})\eta$$

Randomized algorithm

Algorithm 3 A randomized robust and consistent algorithm

1: **if** $y \geq b$ **then**
2: Let $k \leftarrow \lfloor \lambda b \rfloor$
3: **else**
4: Let $k \leftarrow \lceil b/\lambda \rceil$
5: **end if**
6: Define distribution q s.t. $\forall 1 \leq i \leq k$, $q_i \propto (1 - 1/b)^{-i}$.
7: Buy at the start of day j sampled from q.

- Competitive ratio: $\min\{\frac{1}{1 - e^{-(\lambda - 1/b)}}, \frac{\lambda}{(1 - e^{-\lambda})}(1 + \frac{\eta}{OPT})\}$.
- $(\frac{1}{1 - e^{-(\lambda - 1/b)}})$-robust.
- $(\frac{\lambda}{1 - e^{-\lambda}})$-consistent.

Analysis –proof outline.

- A case by case study again, very similar to proof of deterministic case.
- Expectations are computed directly on the distributions q.
- Exponentials appearing from the sums are upper bounded with natural base exponentials.

Randomization helps

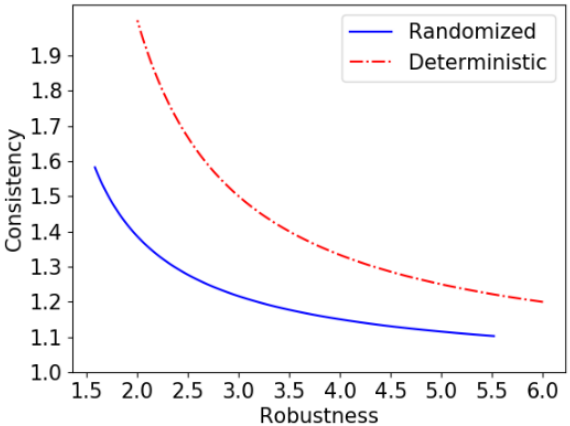


Figure: figure 1 from main paper.

Non-clairvoyant job scheduling with prediction

Non-clairvoyant job scheduling

- Problem:
  - Schedule n jobs on a single machine with no release date.
  - Jobs' length is at least 1.
  - Processing required time x_j of job j is unknown until finished.
  - Any job can be preempted at any time and resumed later at no cost.
  - Objective: minimize sum of flow competition times (*Makespan*).
  - Prediction error: $\eta = \sum_{j=1}^{n} |x_j - y_j|$
- Clairvoyant optimal algorithm: **Shortest Remaining Job First (SRJF)**
  - On any new arrival start processing the job with the least remaining processing time.
  - 1-competitive.
- Optimal algorithm in the non-clairvoyant case: **Round-Robin (RR)**
  - All k unfinished jobs run at the machine at equal rates of $1/k$.
  - 2-competitive (tight).

Algorithm with predictions – design

- We have predictions available.
- RR gets no benefit from predictions.
- Our worst case should then be like RR (2-comp).
- If predictions are perfect we are in the clairvoyant case!
  - We should be then able to perform as good as SRJF.
- Enter **Shortest Predicted Job First (SPJF)**.
  - Factor n off with worst case bad predictions.

Preferential Round Robin

- **Preferential Round Robin**
  - Run RR and SPJF in parallel!
  - SPJF at a rate $1-\lambda$.
  - RR at a rate λ
- Competitive ratio: $\min\{\frac{1}{1-\lambda}\left(1+\frac{2\eta}{n}\right), \frac{2}{\lambda}\}$.
- $\frac{2}{\lambda}$-robust.
- $\frac{2-\lambda}{2-2\lambda}$-consistent. (requires detailed analysis)

Analysis

Definition

An algorithm is *monotonic* if given two instances handling processing times $(x_1, \ldots, x_n)$ and $(x'_1, \ldots, x'_n)$ s.t. $x_i \leq x'_i \ \forall i$, then the objective function f satisfies $f(x_1, \ldots, x_n) \leq f(x'_1, \ldots, x'_n)$.

Lemma 1

Given two monotonic algorithms A and B with competitiveness α and β (respectively) for the minimum makespan problem with preemptions, and a parameter $\lambda \in (0, 1)$, one can obtain a competitive ratio of $\min\{\frac{\alpha}{\lambda}, \frac{\beta}{1-\lambda}\}$

Analysis

Proof of Lemma 1.

By running A and B in parallel, each unit of time is split among the two in a $(\lambda, 1-\lambda)$ split. This would increase the makespan of each one alone by the respective factors: $1/\lambda$ and $1/(1-\lambda)$.

The execution of any algorithm only decreases the remaining processing time of jobs. By monotonicity, this may only reduce the makespan of the other algorithm.

Thus both bounds hold simultaneously. □

Analysis - Competitiveness of SPJF

Lemma 2 Competitiveness of SPJF

The SPJF algorithm has competitive ratio at most $(1 + \frac{2\eta}{n})$

Proof. Assume w.l.o.g. $x_1 \leq \ldots \leq x_n$ For any (i, j) let $d(i, j)$ be the amount of job i executed before completion of job j.

- If $i < j$ and $y_i < y_j \Rightarrow d(i, j) + d(j, i) = x_i + 0$,
- If $i < j$ and $y_i \geq y_j \Rightarrow d(i, j) + d(j, i) = 0 + x_j$

$$ALG = \sum_{j=1}^{n} x_j + \sum_{(i,j):i<j} (d(i, j) + d(j, i))$$

$$= \sum_{j=1}^{n} x_j + \sum_{\substack{(i,j):i<j \\ y_i<y_j}} x_i + \sum_{\substack{(i,j):i<j \\ y_i \geq y_j}} x_j = \sum_{j=1}^{n} x_j + \sum_{(i,j):i<j} x_i + \sum_{\substack{(i,j):i<j \\ y_i \geq y_j}} (x_j - x_i)$$

Analysis - Competitiveness of SPJF

$$
\begin{aligned}
ALG &\leq \sum_{j=1}^{n} x_j + \sum_{(i,j):i<j} x_i + \sum_{\substack{(i,j):i<j \\ y_i \geq y_j}} (\eta_i + \eta_j) \\
&= OPT + \sum_{\substack{(i,j):i<j \\ y_i \geq y_j}} (\eta_i + \eta_j) \\
&\leq OPT + (n-1)\eta
\end{aligned}
$$

$$
x_j \geq 1 \, \forall j \Rightarrow OPT \geq \frac{n(n+1)}{2}
$$

$$
\Rightarrow \frac{ALG}{OPT} \leq 1 + \frac{2(n-1)\eta}{n(n+1)} < 1 + \frac{2\eta}{n} \qquad \square
$$

Analysis

Theorem: Competitiveness of preferential RR.

The preferential RR algorithm with $\lambda \in (0,1)$ has competitive ratio at most $\min\{\frac{1}{1-\lambda}\left(1+\frac{2\eta}{n}\right), \frac{2}{\lambda}\}$. In particular, it is $\frac{2}{\lambda}$-robust and $\frac{1}{1-\lambda}$-consistent.

Proof.

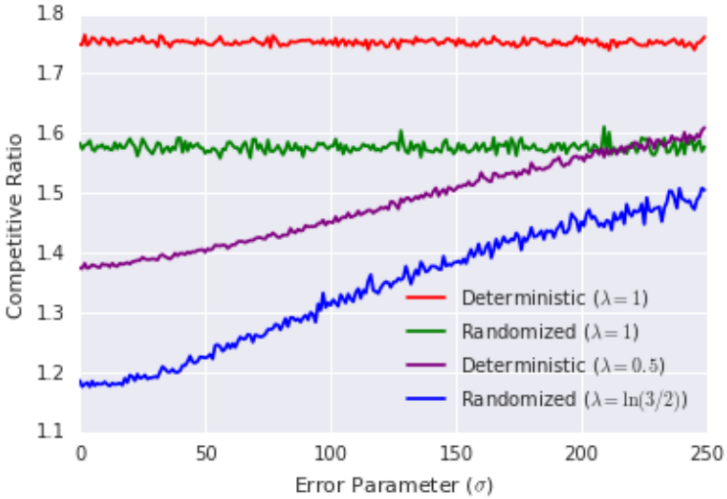
- Recall for SPJF, $c = 1 + \frac{2\eta}{n}$ and for RR $c = 2$.
- Combining both algorithms and apply the result from Lemma 1. □.

Theorem

The competitive ratio of preferential RR under perfect prediction is at most $\frac{2-\lambda}{2-2\lambda}$

Proof ommited.

Skii rental – Experimental results



Non-clairvoyant scheduling –Experimental results

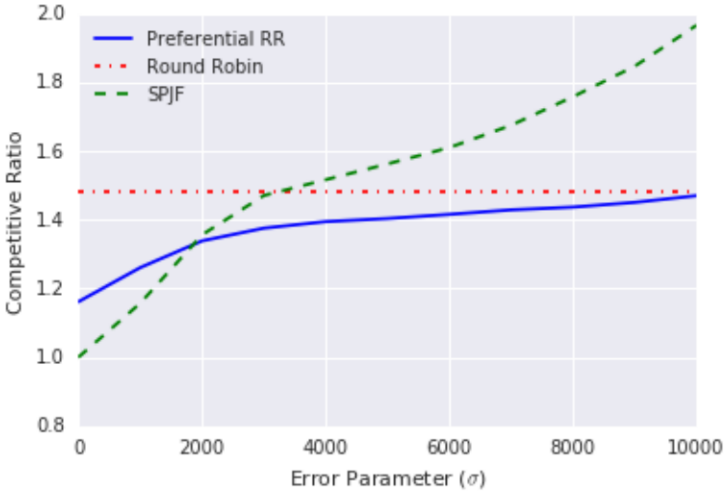


Figure: Non-clairvoyant Scheduling

Conclusions

Conclusions

Conclusions

- Interesting examples of extending online algorithms when predictions are available.
- No systematic approach but a key idea: use a hyperparameter to represent the level of trust in the prediction accuracy.
- How to use the predictions is case-dependent.
- At least in the Skii rental case, randomization helps.

Thank You

Thank You for listening!

Questions?