List Update Problem With Untrusted Advice

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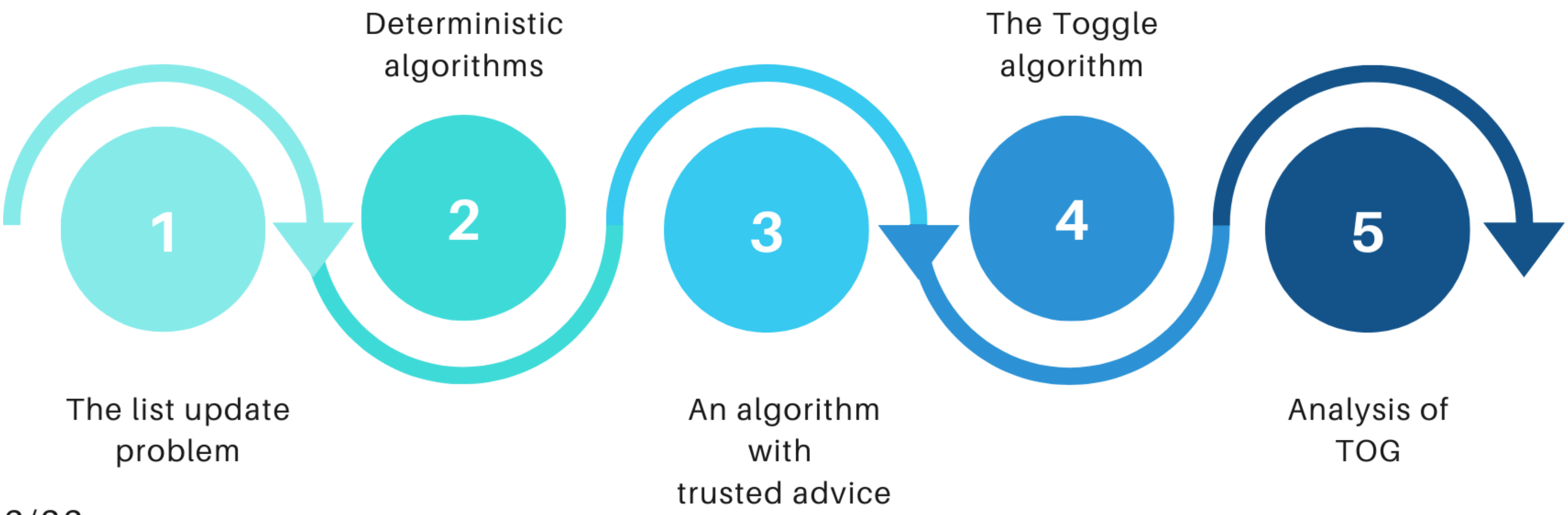
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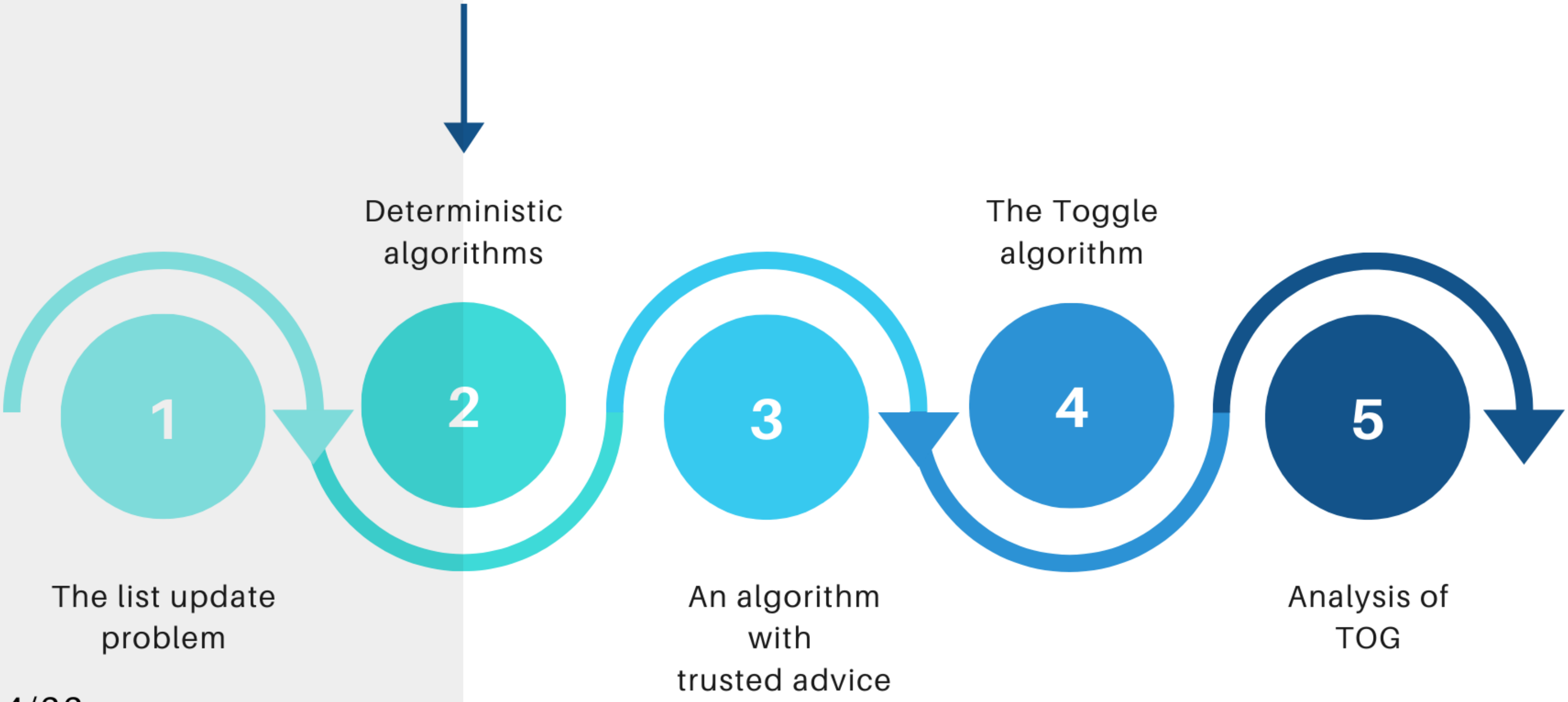
The list update problem

0	1	2	3	4
17	25	31	13	2

1. **Access**
2. **Deletion**
3. **Insertion**

$$\sigma = \sigma_1, \sigma_2, \ldots, \sigma_m$$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MTF

MTF2

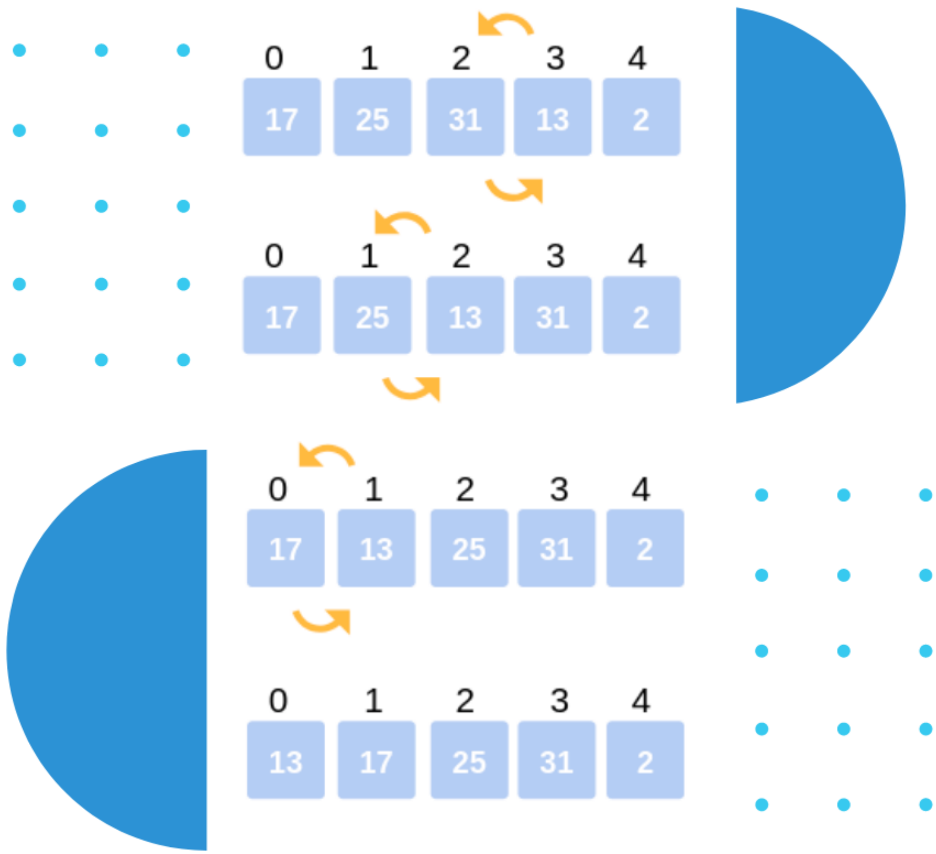
TIMESTAMP

5/22

DETERMINISTIC ALGORITHMS

MTF

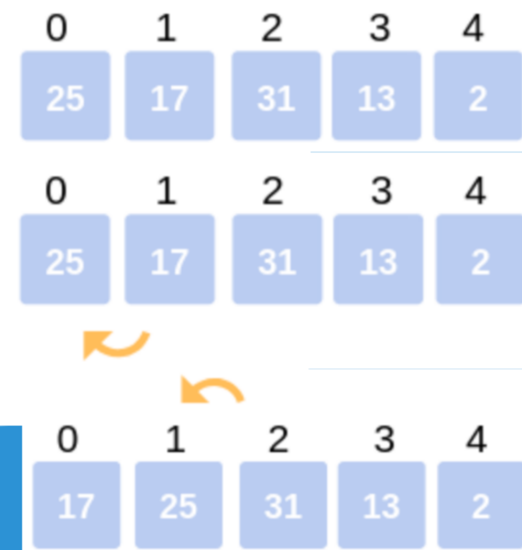
At every step, the algorithm moves the requested item x to the front of the list.
The algorithm is **2-competitive**.



TIMESTAMP

The algorithm inserts the requested item x in front of the first item from the head of the list that precedes x and has been requested at most one time since the last request to x.

The Timestamp algorithm is **2-competitive**.



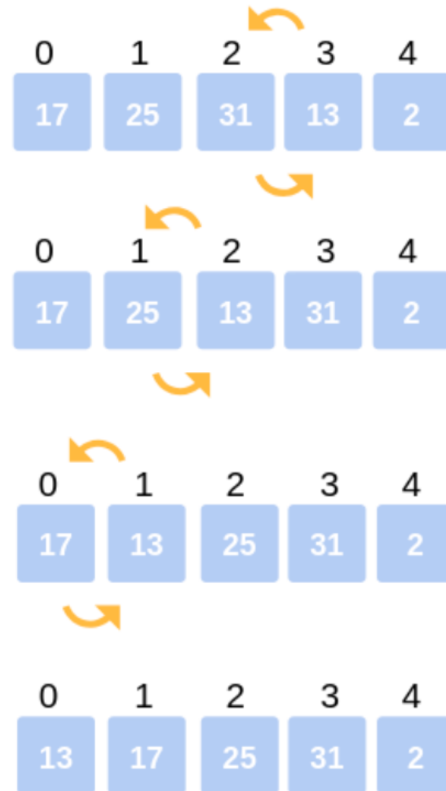
$$\sigma = 17, 25, 17, 2, 31$$

MTF2

MTFE & MTFO

MTFO (resp. MTFE) moves a requested item x to the front on every odd (resp. even) request to x.

The algorithm is **2.5-competitive**.



MTF2

The competitive ratio of MTFO is at least 2.5.

$$\sigma = \langle (a_1, a_2, ..., a_l, a_1^3, a_2^3, ..., a_l^3, a_l, a_{l-1}, ..., a_1, a_l^3, a_{l-1}^3, ..., a_1^3)^m \rangle$$

MTFO :

$$[\overset{1}{a_1} \dots \overset{1}{a_l}] \xrightarrow[l^2/2+o(l^2)]{a_1 \dots a_l} [\overset{0}{a_l} \dots \overset{0}{a_1}] \xrightarrow[2l^2+o(l^2)]{a_1^3 \dots a_l^3} [\overset{1}{a_l} \dots \overset{1}{a_1}] \xrightarrow[l^2/2+o(l^2)]{a_l \dots a_1} [\overset{0}{a_1} \dots \overset{0}{a_l}] \xrightarrow[2l^2+o(l^2)]{a_l^3 \dots a_1^3} [\overset{1}{a_1} \dots \overset{1}{a_l}]$$

OPT :

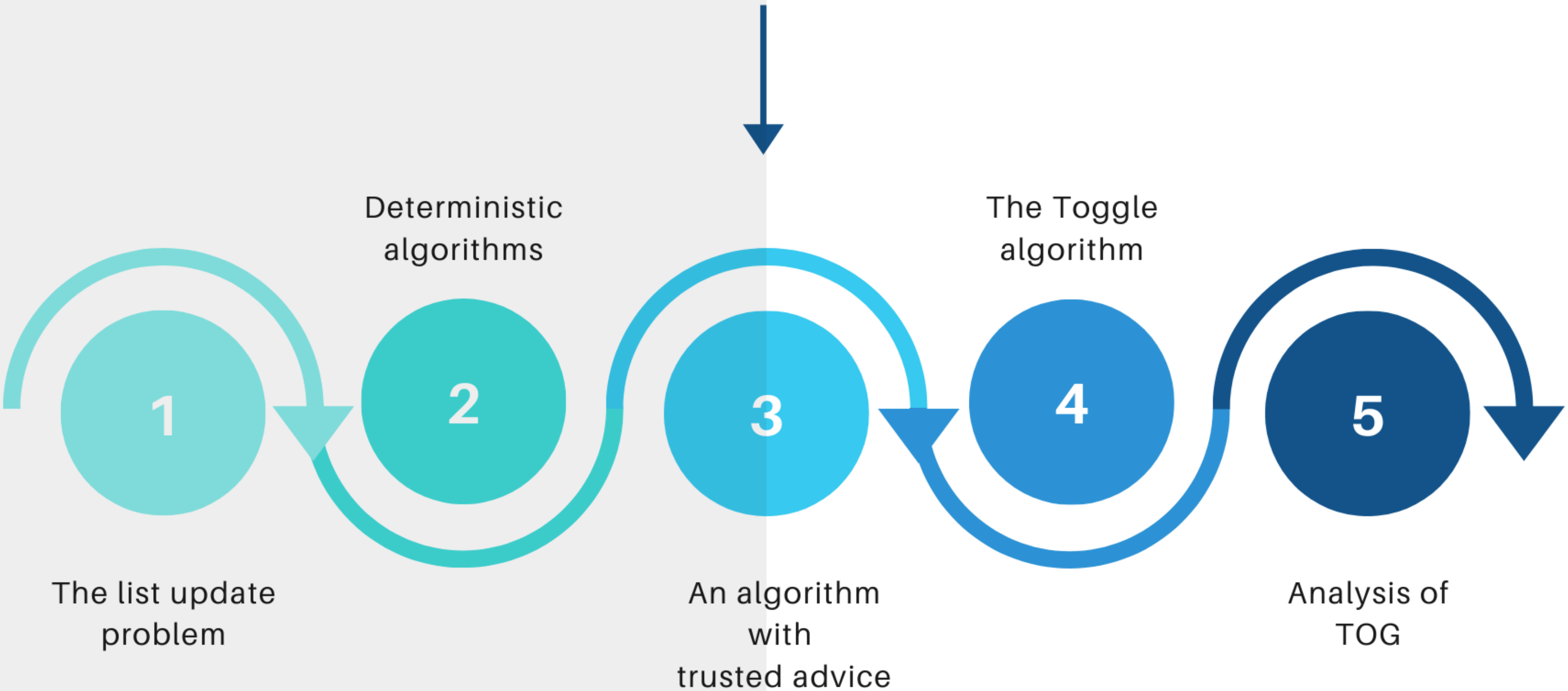
$$[a_1 \dots a_l] \xrightarrow[l^2/2+o(l^2)]{a_1 \dots a_l} [a_1 \dots a_l] \xrightarrow[l^2/2+o(l^2)]{a_1^3 \dots a_l^3} [a_l \dots a_1] \xrightarrow[l^2/2+o(l^2)]{a_l \dots a_1} [a_l \dots a_1] \xrightarrow[l^2/2+o(l^2)]{a_l^3 \dots a_1^3} [a_1 \dots a_l]$$

MTF2

The competitive ratio of MTFO is at most 2.5.

Phase	MTF2	OPT'	ratio
$x^j yy$	≤ 2	1	≤ 2
$x^j (yx)^{2i} yy$	$\leq 3i+2$	$2i+1$	< 2
$x^j (yx)^{2i-2} yxyy$	$\leq 3(i-1)+4$	$2i$	≤ 2
$x^j (yx)^{2i} x$	$\leq 3i+1$	$2i$	≤ 2
$x^j (yx)^{2i} yxx$	$\leq 3i+3$	$2i+1$	≤ 2
$x^j yxx$	3	1	3

Critical Phase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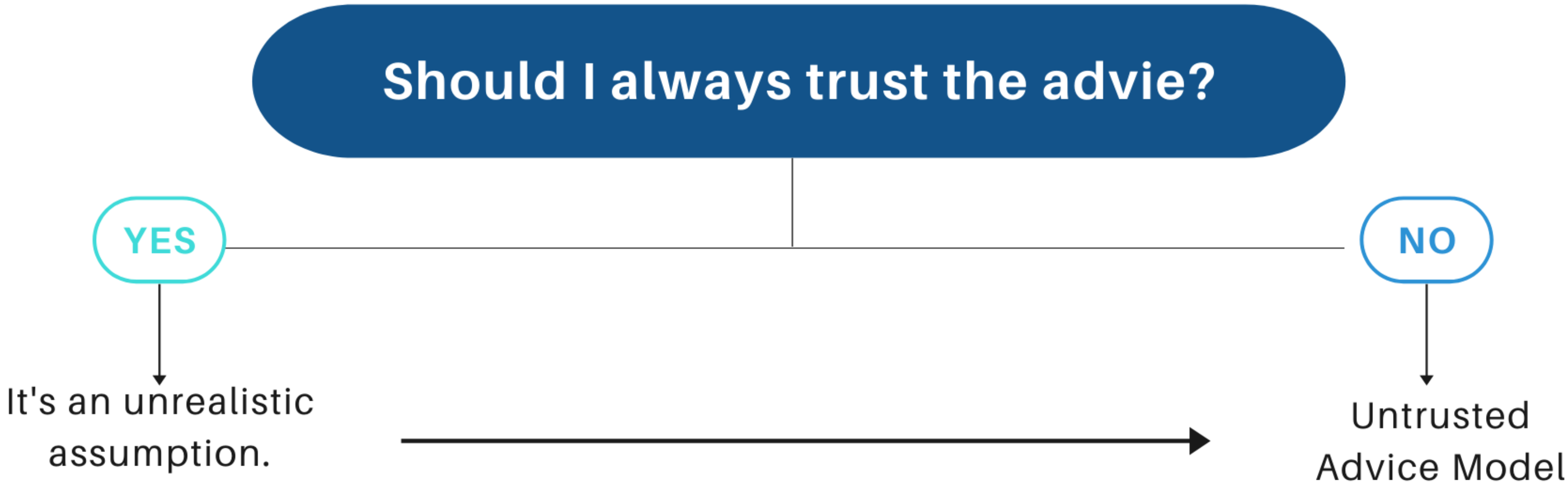
ADVICE MODEL

Under this model, the online algorithm receives partial information about the unknown parts of the input in the form of some bits of advice, generated by a benevolent offline oracle with infinite computational power.

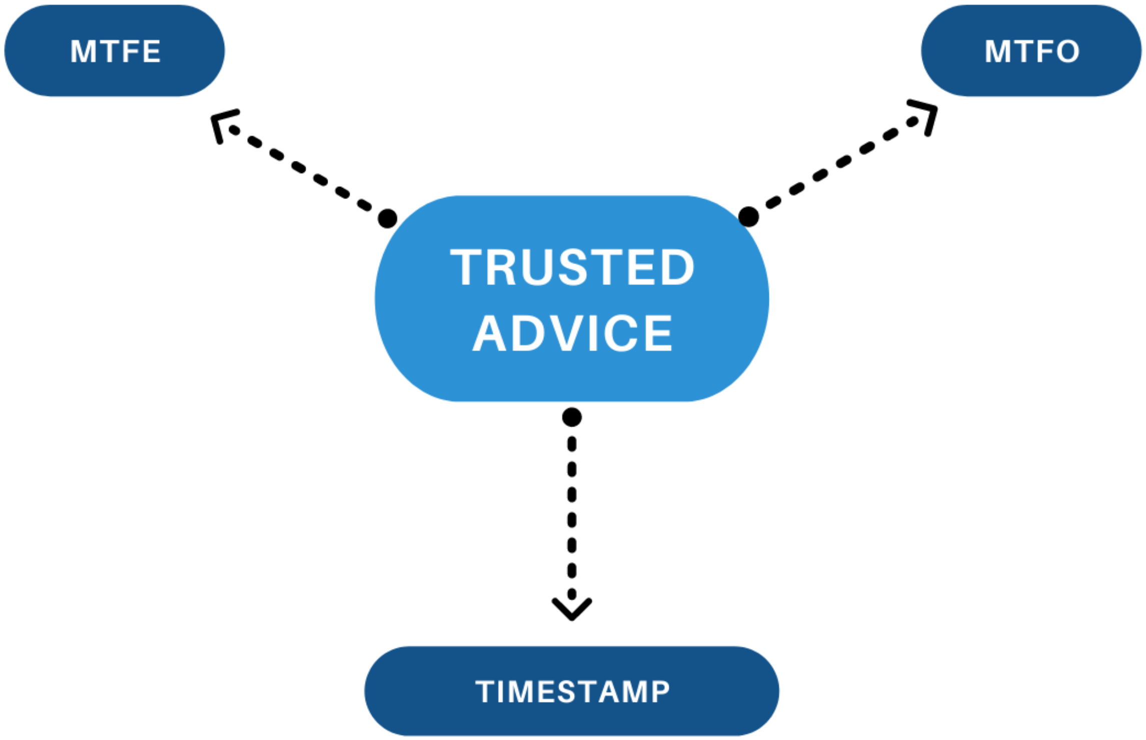
ADVICE MODEL

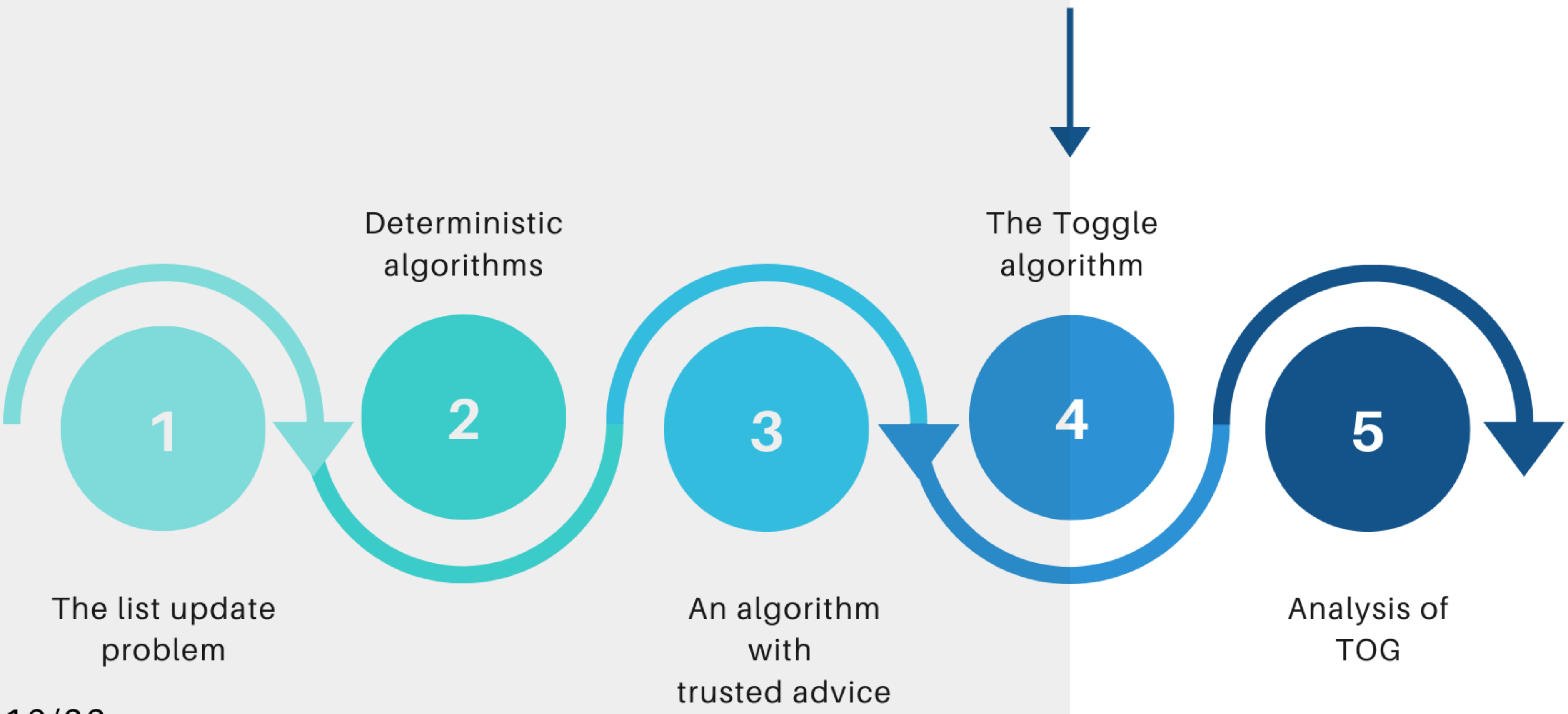


$$\forall \sigma, \forall x, y \in \sigma : MTFO(\sigma_{xy}) + MTFE(\sigma_{xy}) + Timestamp(\sigma_{xy}) \leq 5 \cdot Opt(\sigma_{xy})$$

Phase	AlgMin	AlgMax	Timestamp	Sum (AlgMin + AlgMax + Timestamp)	Opt'	$\frac{\text{Sum}}{\text{Opt}'}$
$x^j yy$	1	2	2	5	1	5
$x^j (yx)^{2i} yy$	$\leq 3i + 1$	$\leq 3i + 2$	$2 \cdot 2i = 4i$	$\leq 10i + 3$	$2i + 1$	< 5
$x^j (yx)^{2i-2} yxyy$	$\leq 3(i-1) + 1$ $+ \text{AlgMin}(\langle xyy \rangle)$	$\leq 3(i-1) + 1$ $+ \text{AlgMax}(\langle xyy \rangle)$	$2(2i-1)$ $= 4i - 2$	$\leq 6(i-1) + 2 + 4$ $+(4i-2) = 10i - 2$	$2i$	< 5
$x^j (yx)^{2i} x$	$\leq 3i$	$\leq 3i + 1$	$2 \cdot 2i - 1$ $= 4i - 1$	$\leq (6i + 1) + (4i - 1)$ $= 10i$	$2i$	≤ 5
$x^j (yx)^{2i-2} yxx$	$\leq 3(i-1)$ $+ \text{AlgMin}(\langle yxx \rangle)$	$\leq 3(i-1)$ $+ \text{AlgMax}(\langle yxx \rangle)$	$2 \cdot (2i-1) - 1$ $= 4i - 3$	$\leq 6(i-1) + 4$ $+(4i-3) = 10i - 5$	$2i - 1$	≤ 5

1.6-COMPETITIVE

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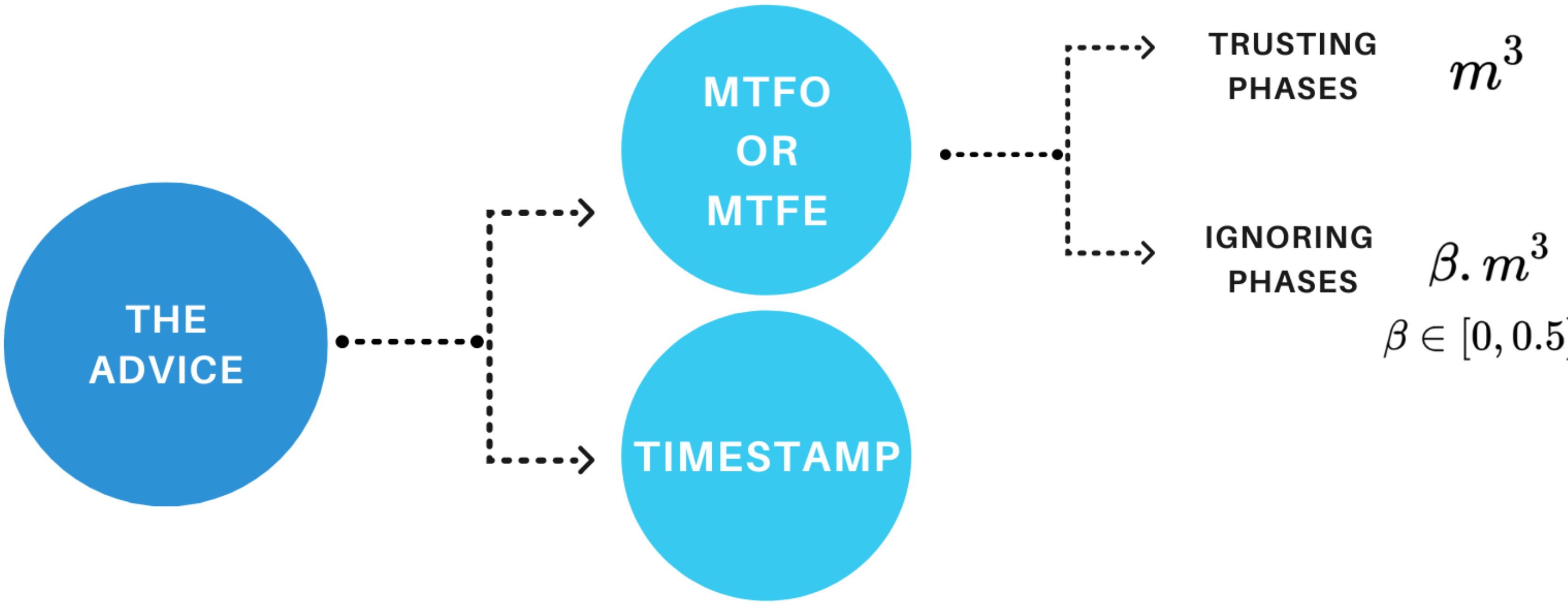
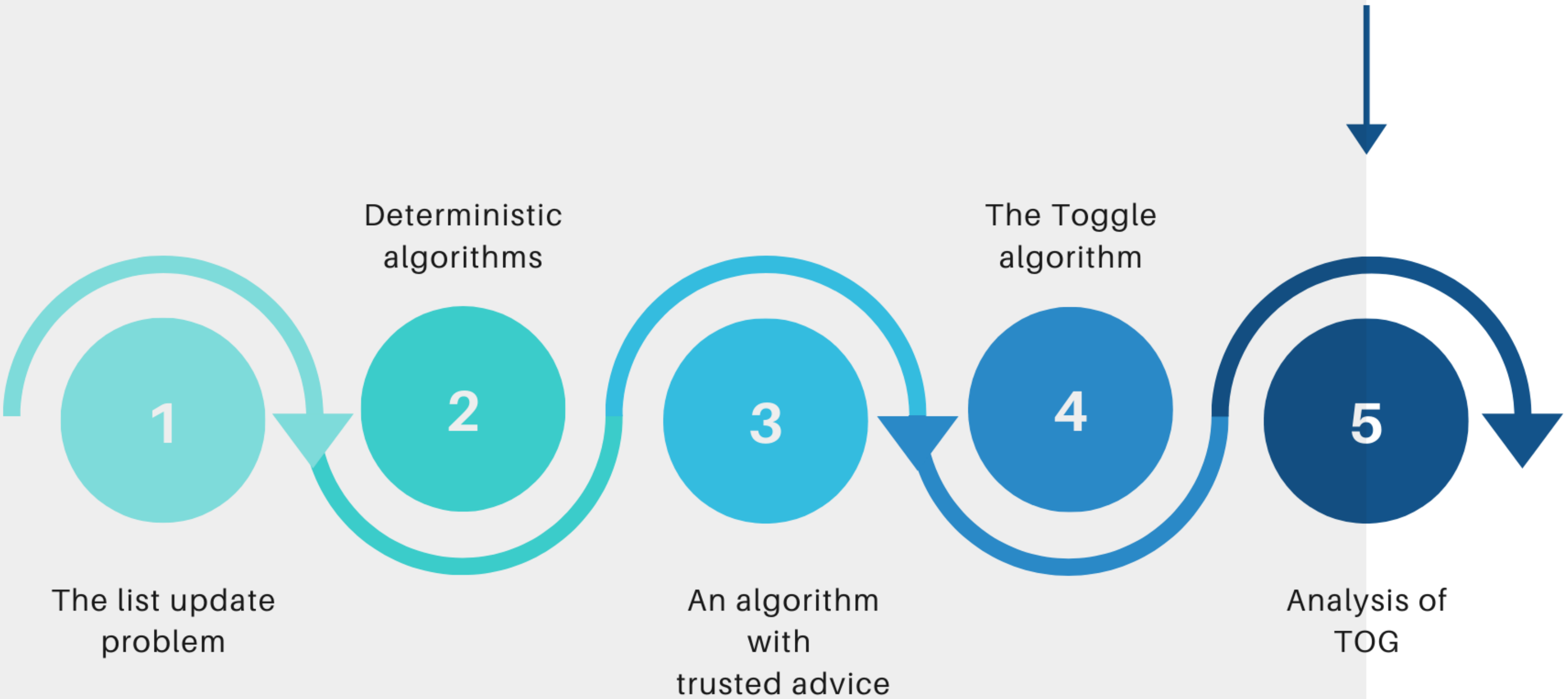
THE TOGGLE ALGORITHM

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The cost of phases

TRUSTING

$$m^3(1 + 1/m), m^3(1 + 1/m + 1/m^2)$$

IGNORING

$$(\beta m^3, \beta m^3(1 + 1/m^2))$$

$$k.\, m^3.\, (1 + \beta + 3/m)$$

COMPETITIVE RATIO OF TOG

ADVICE

TRUSTED

$$5/3 + \frac{5\beta}{6 + 3\beta}$$

UNTRUSTED

$$2.5 - \frac{5\beta}{8 + 10\beta}$$

Undiscussed Questions

Parameter Optimization (Beta)

Expected Error

The cost thresholds of TOG's phases

References

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THANK YOU

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